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Lorentzian Goldstone modes shared among photons and gravitons

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Lorentz violation source of emergent theories

- Emergent Electro-Magnetism
- Emergent Yang-Mills theories
- Emergent Tensor gravity

- Introduce potential

$$V = \lambda\left(A_\mu^2 - n_\mu^2 M^2\right)^2$$

Symmetry violation produces Higgs mode with mass proportional to M and vector Goldstone

M, n_μ Lorentz violation scale and direction In the space-time

If n_μ is time-like $SO(1,3) \to SO(3)$

If n_μ is space-like $SO(1,3) \to SO(1,2)$

Hamiltonian is unbouned from below if following is not satisfied

$$A_\mu^2 = n_\mu^2 M^2$$

Alternatively we take σ-model limit by $\lambda \to \infty$

Emergent vector field theory

- Non-Linear sigma model for Abelian Vector field

$$L_M = -\frac{1}{4} {F_{\alpha\beta}}^2 \qquad A_\mu^2 = M^2 n_\mu^2$$

Expansion into Goldstone modes

$$A_\mu = a_\mu + \frac{n_\mu}{n_\mu^2}(n^\nu A_\nu) \qquad n_\mu a^\mu = 0$$

- Non-Linear sigma model for Yang-Mills field

$$L = -\frac{1}{4}(F_{\alpha\beta}^i)^2 \qquad (A_\mu^i)^2 = Mn^2$$

$$A_\mu^i = a_\mu^i + \frac{n_\mu^i}{n^2}(nA) \qquad a_\mu^i n^{\mu i} = 0 \qquad n_\mu^i = n_\mu\, s^i$$

Emergent Tensor gravity

L_{fp} $\qquad H_{\alpha\beta}^2 = M^2 n_{\alpha\beta}^2$

$n_{00} \neq 0\,, \quad SO(1,3) \to SO(3)$

$n_{ij} \neq 0\,, \quad i = j, \quad SO(1,3) \to SO(1,2)$

$n_{ij} \neq 0\,, \quad i \neq j, \quad SO(1,3) \to SO(1,1)$

$n_{0j} \neq 0\,, \quad SO(1,3) \to SO(2)$

$$H_{\mu\rho} = h_{\mu\rho} + \frac{n_{\mu\rho}}{n^2}(nH) \qquad h_{\mu\rho} n^{\mu\rho} = 0$$

Lorentz breaking interactions

- LB self interaction between vector goldstone

$$-\frac{1}{4}\frac{n^2}{M}f^{\alpha\beta}(n_\alpha\partial_\beta - n_\beta\partial_\alpha)a^2$$

- If also source included, LB with a source current

$$-\frac{a^2}{2M}n^\alpha J_\alpha$$

Lorentz breaking interactions for Yang-Mills vectors

- LB 3-vector Goldstone self interaction

$$-\frac{1}{4}\frac{n^2}{M}\boldsymbol{f}^{\alpha\beta}(\boldsymbol{n}_\alpha\,\partial_\beta-\boldsymbol{n}_\beta\,\partial_\alpha)\boldsymbol{a}^2$$

- LB 4-vector self interaction

$$-ig\frac{1}{4}\frac{n^2}{M}\boldsymbol{f}^{i\alpha\beta}([\boldsymbol{n}_\alpha\,\boldsymbol{a}_\beta]-[\boldsymbol{n}_\beta\,\boldsymbol{a}_\alpha])\boldsymbol{a}^2$$

- And LB interaction with a current

$$-\frac{\boldsymbol{a}^2}{2M}n^{i\alpha}J^i_\alpha$$

Lorentz breaking interactions

- LB 3-graviton self interaction

$$-\frac{n^2}{M}h^2 n^{\alpha\beta}(\partial_\alpha \partial^\gamma h_{\beta\gamma} - \frac{1}{2}\partial_\alpha \partial_\beta h_{tr})$$

- LB interaction with a gravity source energy-momentum tensor

$$\frac{n^2}{2M_p M}h^2 n^{\alpha\beta}T_{\alpha\beta}$$

What we learned so far

- Spontaneous Lorentz invariance violation gives a birth to an emergent theories.
- There is no Physical Lorentz violation In the Goldstone gauge sector nor in the interaction with source fields.
- Gauge invariance protects Lorentz invariance.

Emergent electrogravity model

We discus Lorentz violation for the vector and tensor fields in the same framework and also include scalar as a source for both of them

$$L_{tot} = L_H + L_A + L_\varphi + L_{int}$$

$$H^2_{\alpha\beta} = M^2_H n^2_{\alpha\beta} \qquad A^2_\mu = M^2_A n^2$$

- Vector field is a carrier of the EM interaction and the source for gravity same time

Potential that respects symmetries of the both condition.

$$V(H,A) = \lambda_A(A_\alpha^2 - \mathrm{n}^2 M_A^2)^2 + \lambda_H\left(H_{\alpha\beta}^2 - n_{\alpha\beta}^2 M_A^2\right)^2 + \lambda_{AH} A_\alpha^2 H_{\alpha\beta}^2$$

$$\lambda_{A,H} > 0\,, \qquad \lambda_A \lambda_H > \lambda_{AH}/4$$

Theory is generally unstable, but in the limit $\lambda_{A,H} \to \infty$ stability is restored and we arrive to the length fixing conditions.

Symmetry violation patterns are

- Minimally broken symmetry, when vacuum evolves on same components for fields, SO(1,3) → SO(1,2) or SO(3)
- When SO(1,3) → SO(1,1) or SO(2)
- Fully broken Lorentz symmetry

Using length fixing condition we apply expansion into Goldstone modes for vector and tensor fields around the corresponding vacuum states

$$H_{\mu\rho} = n_{\mu\rho} M_H + h_{\mu\rho} - \frac{n^2 h^2}{2M_H} n_{\mu\rho} + O(\frac{1}{M_H^2})$$

$$A_\mu = \mathrm{n}_\mu M_A + a_\mu - \frac{\mathrm{n}^2 a^2}{2M_A} \mathrm{n}_\mu$$

After applying this expansions from L_{int} we get big unphysical terms

$$\frac{M_H}{M_P} n_{\alpha\beta} T^{\alpha\beta} \quad \text{and} \quad e\frac{M_A}{M_P} H_{\alpha\beta} \mathrm{n}^\alpha J^\beta$$

We need to redefine source fields to eliminate not-physical terms.

$$\delta\varphi = -ieM_A(\mathrm{n}x) + \frac{M_H}{M_P} n^{\alpha\beta} x_\alpha \partial_\beta \varphi$$

$$\delta a_\mu = \frac{M_H}{M_P} n^{\alpha\beta} x_\alpha \partial_\beta a_\mu + \frac{M_H}{M_P} n_{\mu\alpha} a^\alpha$$

This is basically sum of U(1) gauge and diff. transformations and we also clearly see that they commute.

We get 2 type of characteristic interactions for this model.

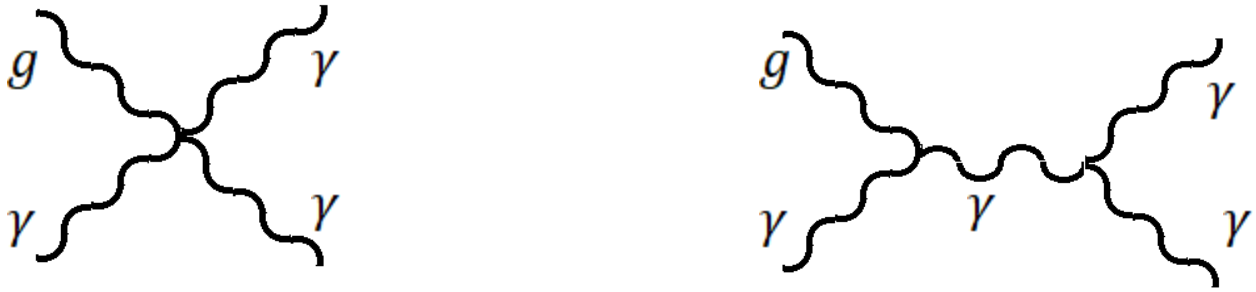
The first appears because vector also evolves VEV and is suppressed by corresponding mass scale

$$\frac{n^2 h^{\alpha\beta}}{M_P M_A}\left(f_\alpha^\gamma \partial_{\beta\gamma} a^2 - \mathrm{n}_\alpha \mathrm{J}_\beta - \eta_{\alpha\beta}\left(\frac{1}{4} f^{\gamma\rho} \partial_{\beta\gamma} a^2 - \mathrm{n}_\rho J^\rho\right)\right)$$

The second one is gravity induced new interaction approximately on par with gravity strength or even stronger if $M_H \gg M_A$

$$\frac{n^2 M_H}{M_P M_A} n^{\alpha\beta} a_\alpha a_\beta \mathrm{n}^\rho (\partial^\mu f_{\mu\rho} - J_\rho)$$

Photon graviton conversion



Elastic photon-scalar scattering

Is length fixing condition only gauge fixing?

We need more profound physical motivation and calculating only first order effects is not enough. General approach would be to find gauge function that satisfies constraint condition.

For vector field it is $(A_\alpha + \partial_\alpha \omega)^2 = \mathrm{n}^2 M^2$

For tensor field $\left(H_{\alpha\beta} + \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha\right)^2 = n^2 M^2$

Challenges of finding gauge function

Lets make vector field case. For time-like violation

$$(A_\alpha + \partial_\alpha \omega)^2 = \mathrm{M}^2$$

Hamilton Jacobi equation of relativistic particle with mass M moving in the EM field A_α. Taking ω to be the action of the system

$$\omega = \int (M\sqrt{U_\alpha U^\alpha} - U^\alpha A_\alpha) d\tau \qquad\qquad p_\alpha = \partial_\alpha \omega$$

In space-like violation case $(A_\alpha + \partial_\alpha \omega)^2 = -\mathrm{M}^2$ we do not have such correspondence . We can come up with functional

$$\omega = \int (M\sqrt{-U_\alpha U^\alpha} - U^\alpha A_\alpha) d\tau$$

We can find same kind of hints for tensor case as well

$$\left(H_{\alpha\beta} + \partial_\alpha \xi_\beta + \partial_\beta \xi_\alpha\right)^2 = M^2$$

We can fix 3 degrees of freedom to simplify task.

$$H_{0i} = -\partial_0 \xi_i \quad \rightarrow \quad \left(H_{00} + 2\partial_0 \xi_0\right)^2 - 2\left(\partial_i \xi_0\right)^2 = M^2 - H_{ij}^2$$

System this can be compared to, can be given by following action

$$S = \int (\sqrt{M^2 - H_{ij}^2}\sqrt{1 - v_i^2} - H_{00})dt$$

We have other way as well. We can use Goldstone field in the equation

$$(a_\alpha + \partial_\alpha \omega)^2 = \mathrm{n}^2 M^2$$

and take into account $a^\mu \mathrm{n}_\mu = 0$, $\frac{a_\mu^2}{M^2} \ll 1$ and try to find solution in the form series $\omega = \omega_0 + \sum \frac{\omega_k}{M^k}$.

So, we find $\omega_0 = M(n_\mu x^\mu)$, $\omega_1 = -\frac{1}{2}\int a_\nu^2 \, d(n_\mu x^\mu)$

$$\omega_2 = -\int a^\mu \, \partial_\mu \omega_1 d(n_\mu x^\mu)$$

$$\omega_{k+2} = -\int \left(a^\mu \partial_\mu \omega_{k+1} + \frac{1}{2}\sum_{m=1}^{k} \partial_\mu \omega_m \partial^\mu \omega_{k-m} \right) d(n_\mu x^\mu)$$

Summery for electrogravity model

- We see that such SLIV pattern, induces massless vector and tensor Goldstone and pseudo-Goldstone modes some of which can naturally be associated with the physical photon and graviton
- While Photon can consist from pure goldstone modes graviton may need to accommodate pseudo goldstone modes as well.
- The contains a variety of Lorentz and CP T violating couplings, but still Lorentz violation is superficial
- From this standpoint, the only way for physical Lorentz violation to appear would be if the above local invariance is slightly broken at very small distances and that would most probably give some effects proportional to $\frac{M_H}{M_P}$ and/or $\frac{M_A}{M_P}$

Thanks for attention